

Validation of General Purpose CAE, CFD Simulation Code -hFlow-

I. Mesh Generation – gFlow –

gFlow™(Mesh Generation Code)

1. Elliptic mesh generation equation

$$\alpha_{11}x_{\xi\xi} + \alpha_{12}x_{\xi\eta} + \alpha_{13}x_{\xi\phi} + \alpha_{22}x_{\eta\eta} + \alpha_{23}x_{\eta\phi} + \alpha_{33}x_{\phi\phi} = -I^2(Px_{\xi} + Qx_{\eta} + Rx_{\phi})$$

2. Control function(P, Q and R) suggested by Thomas and Middlecoff(1980)

3. Jacobian and coefficient

$$I = x_{\xi}(y_{\eta}z_{\phi} - y_{\phi}z_{\eta}) - x_{\eta}(y_{\xi}z_{\phi} - y_{\phi}z_{\xi}) + x_{\phi}(y_{\xi}z_{\eta} - y_{\eta}z_{\xi})$$

$$\alpha_{11} = \beta_{11}^2 + \beta_{21}^2 + \beta_{31}^2$$

$$\alpha_{12} = \beta_{11}\beta_{12} + \beta_{21}\beta_{22} + \beta_{31}\beta_{32}$$

$$\alpha_{13} = \beta_{11}\beta_{13} + \beta_{21}\beta_{23} + \beta_{31}\beta_{33}$$

$$\alpha_{22} = \beta_{12}^2 + \beta_{22}^2 + \beta_{32}^2$$

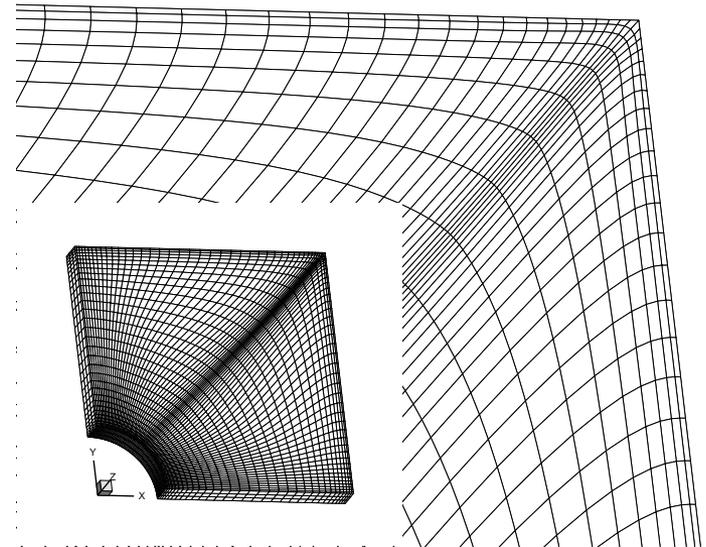
$$\alpha_{23} = \beta_{12}\beta_{13} + \beta_{22}\beta_{23} + \beta_{32}\beta_{33}$$

$$\alpha_{33} = \beta_{13}^2 + \beta_{23}^2 + \beta_{33}^2$$

$$\beta_{11} = y_{\eta}z_{\phi} - y_{\phi}z_{\eta}, \beta_{12} = y_{\phi}z_{\xi} - y_{\xi}z_{\phi}, \beta_{13} = y_{\xi}z_{\eta} - y_{\eta}z_{\xi}$$

$$\beta_{21} = x_{\phi}z_{\eta} - x_{\eta}z_{\phi}, \beta_{22} = x_{\xi}z_{\phi} - x_{\phi}z_{\xi}, \beta_{23} = x_{\eta}z_{\xi} - x_{\xi}z_{\eta}$$

$$\beta_{31} = x_{\eta}y_{\phi} - x_{\phi}y_{\eta}, \beta_{32} = x_{\phi}y_{\xi} - x_{\xi}y_{\phi}, \beta_{33} = x_{\xi}y_{\eta} - x_{\eta}y_{\xi}$$



4. Mesh generation using gFlow™

- High orthogonality
- Desired space control
- High smoothness

II. Convection and Diffusion Scheme

– hFlow –

hFlow™(Heat, Mass & Fluid Flow Analysis Code)

1. u-Momentum Equation

$$\begin{aligned} \frac{\partial}{\partial \xi}(\rho Uu) + \frac{\partial}{\partial \eta}(\rho Vu) + \frac{\partial}{\partial \phi}(\rho Wu) = & - \left(f_{11} \frac{\partial p}{\partial \xi} + f_{21} \frac{\partial p}{\partial \eta} + f_{31} \frac{\partial p}{\partial \phi} \right) \\ & + \frac{\partial}{\partial \xi} \left[\frac{\Gamma}{J} \left(q_{11} \frac{\partial u}{\partial \xi} + q_{12} \frac{\partial u}{\partial \eta} + q_{13} \frac{\partial u}{\partial \phi} \right) \right] \\ & + \frac{\partial}{\partial \eta} \left[\frac{\Gamma}{J} \left(q_{21} \frac{\partial u}{\partial \xi} + q_{22} \frac{\partial u}{\partial \eta} + q_{23} \frac{\partial u}{\partial \phi} \right) \right] \\ & + \frac{\partial}{\partial \phi} \left[\frac{\Gamma}{J} \left(q_{31} \frac{\partial u}{\partial \xi} + q_{32} \frac{\partial u}{\partial \eta} + q_{33} \frac{\partial u}{\partial \phi} \right) \right] \end{aligned}$$

2. Energy Equation

$$\begin{aligned} \frac{\partial}{\partial \xi}(\rho UH) + \frac{\partial}{\partial \eta}(\rho VH) + \frac{\partial}{\partial \phi}(\rho WH) \\ = \frac{\partial}{\partial \xi} \left[\frac{\Gamma_h}{J} \left(q_{11} \frac{\partial h}{\partial \xi} + q_{12} \frac{\partial h}{\partial \eta} + q_{13} \frac{\partial h}{\partial \phi} \right) \right] + \frac{\partial}{\partial \eta} \left[\frac{\Gamma_h}{J} \left(q_{21} \frac{\partial h}{\partial \xi} + q_{22} \frac{\partial h}{\partial \eta} + q_{23} \frac{\partial h}{\partial \phi} \right) \right] \\ + \frac{\partial}{\partial \phi} \left[\frac{\Gamma_h}{J} \left(q_{31} \frac{\partial h}{\partial \xi} + q_{32} \frac{\partial h}{\partial \eta} + q_{33} \frac{\partial h}{\partial \phi} \right) \right] + \frac{\partial}{\partial \xi} \left[\frac{\Gamma_k}{J} \left(q_{11} \frac{\partial k}{\partial \xi} + q_{12} \frac{\partial k}{\partial \eta} + q_{13} \frac{\partial k}{\partial \phi} \right) \right] \\ + \frac{\partial}{\partial \eta} \left[\frac{\Gamma_k}{J} \left(q_{21} \frac{\partial k}{\partial \xi} + q_{22} \frac{\partial k}{\partial \eta} + q_{23} \frac{\partial k}{\partial \phi} \right) \right] + \frac{\partial}{\partial \phi} \left[\frac{\Gamma_k}{J} \left(q_{31} \frac{\partial k}{\partial \xi} + q_{32} \frac{\partial k}{\partial \eta} + q_{33} \frac{\partial k}{\partial \phi} \right) \right] \\ + \Phi \end{aligned}$$

3. Continuity Equation

$$\frac{\partial}{\partial \xi}(\rho U) + \frac{\partial}{\partial \eta}(\rho V) + \frac{\partial}{\partial \phi}(\rho W) = 0$$

4. Total Enthalpy

$$\begin{aligned} H &= E + p \\ &= \rho \left(e + \frac{1}{2} [u^2 + v^2 + w^2] \right) + p \end{aligned}$$

hFlowTM(Code Validation for Convection Scheme)

1. Validation from

- U. Ghia, K. N. Ghia, and C. T. Shin(1982)

2. Boundary condition

- Lid driven cavity
- $Re=10^4$

3. Numerical scheme

Convection scheme

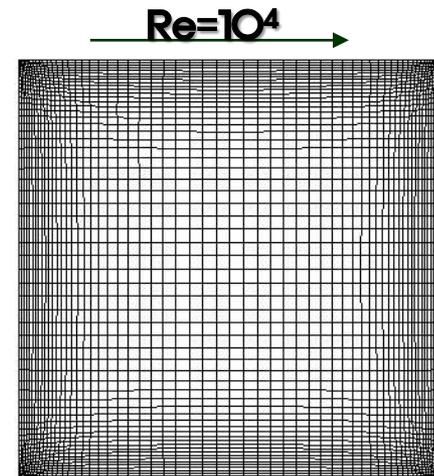
- 1st Order Upwind
- 2nd Order Upwind
- QUICK

Flux splitting scheme

- Liou et al. (1990)

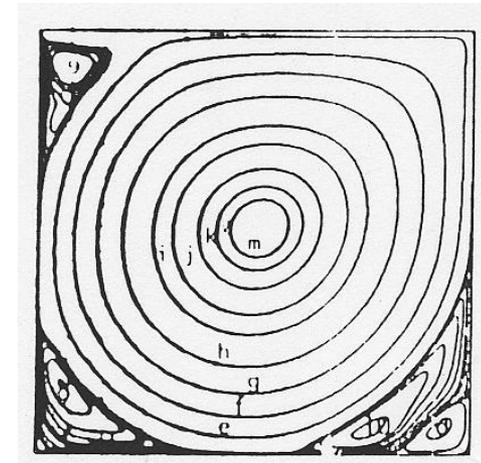
Pressure & velocity linkage

- SIMPLE



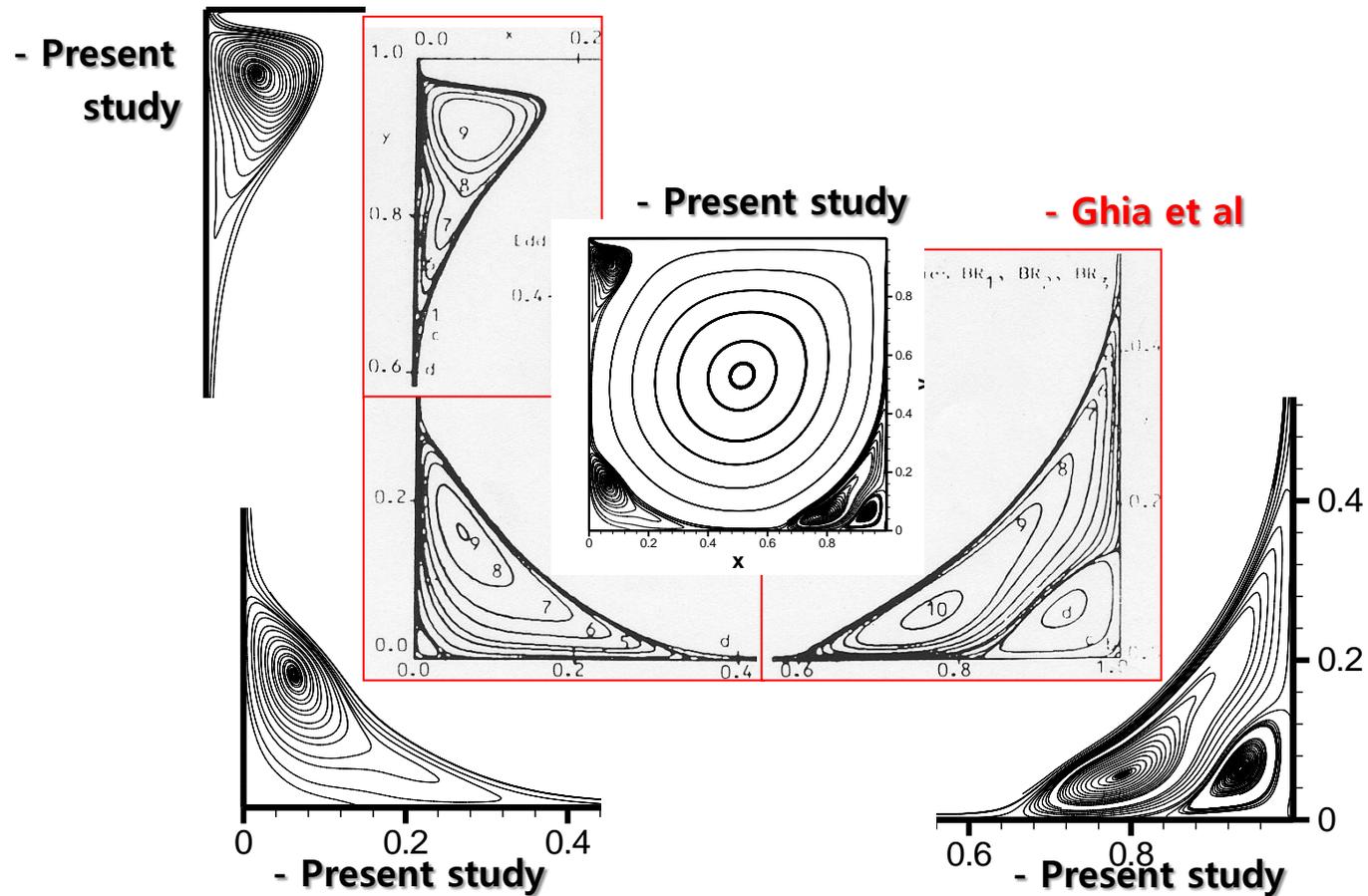
4. Mesh generation

- Numerical mesh(80 by 80)



5. Streamline from Ghia et al.

hFlow™(Code Validation for Convection Scheme)



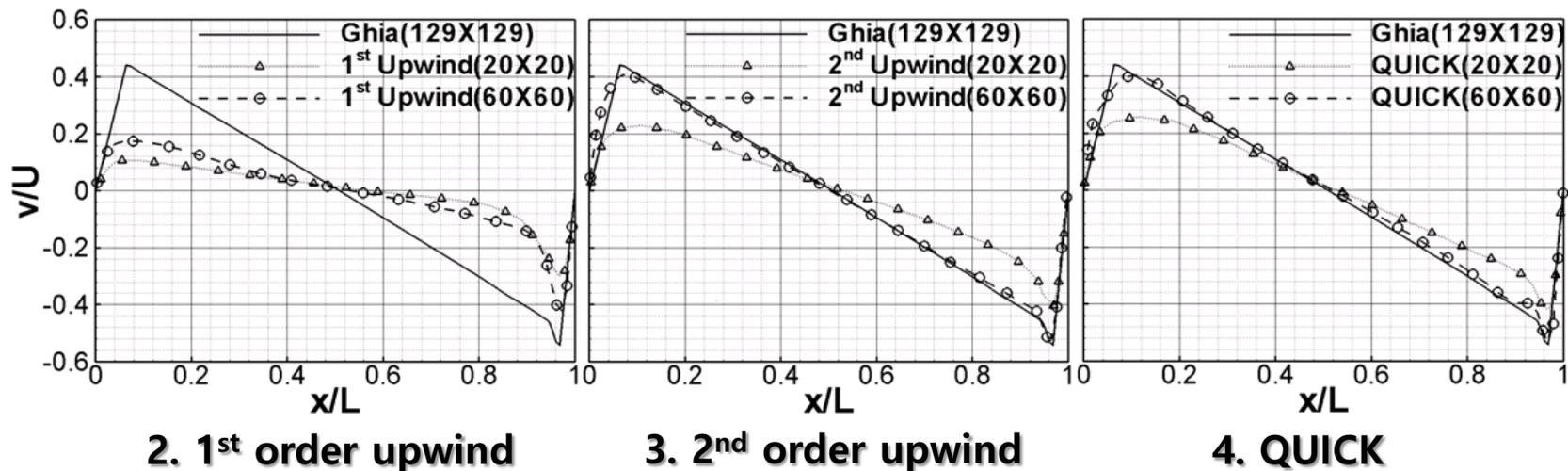
1. Comparison of streamline pattern.

- Present study : QUICK
- Ghia et al. : 3rd order upwind

hFlowTM(Code Validation for Convection Scheme)

- Vertical velocity distribution comparison of present study results with those of Ghia et al.* at the mid-height of lid driven square cavity for various convection scheme and mesh size.

* U. Ghia, K. N. Ghia, and C. T. Shin, "High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method," J. Compt. Phys. 48, 387, (1982).



III. Turbulence Scheme – hFlow –

hFlow™(Turbulence Flow Analysis Code)

1. Turbulence Kinetic Energy and Dissipation Rate Equation

$$\begin{aligned} \frac{\partial \rho \Phi}{\partial t} + \frac{\partial}{\partial \xi}(\rho U \Phi) + \frac{\partial}{\partial \eta}(\rho V \Phi) + \frac{\partial}{\partial \phi}(\rho W \Phi) = & \frac{\partial}{\partial \xi} \left[\frac{\Gamma_{\Phi}}{J} \left(q_{11} \frac{\partial \Phi}{\partial \xi} + q_{12} \frac{\partial \Phi}{\partial \eta} + q_{13} \frac{\partial \Phi}{\partial \phi} \right) \right] \\ & + \frac{\partial}{\partial \eta} \left[\frac{\Gamma_{\Phi}}{J} \left(q_{21} \frac{\partial \Phi}{\partial \xi} + q_{22} \frac{\partial \Phi}{\partial \eta} + q_{23} \frac{\partial \Phi}{\partial \phi} \right) \right] \\ & + \frac{\partial}{\partial \phi} \left[\frac{\Gamma_{\Phi}}{J} \left(q_{31} \frac{\partial \Phi}{\partial \xi} + q_{32} \frac{\partial \Phi}{\partial \eta} + q_{33} \frac{\partial \Phi}{\partial \phi} \right) \right] \\ & + R_1 + R_2 \end{aligned}$$

2. For Turbulence Kinetic Energy Equation

$$R_1 = \Pi \quad R_2 = -\rho \varepsilon = - \left(\frac{C_{\mu} \rho^2 k^*}{\mu_t} \right) k$$

3. For Turbulence Dissipation Rate Equation

$$R_1 = \frac{C_1 \varepsilon \Pi}{k} \quad R_2 = C_2 \rho \frac{\varepsilon^2}{k} = - \left(\frac{C_2 \rho \varepsilon^*}{k^*} \right) \varepsilon$$

4. Production and eddy viscosity

$$\Pi = \tau_{ij} \frac{\partial \langle u_i \rangle}{\partial x_j} = \left[\mu_t \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) - p \delta_{ij} \right] \frac{\partial \langle u_i \rangle}{\partial x_j} \quad \mu_t = \frac{C_{\mu} f_{\mu} \rho k^2}{\varepsilon}$$

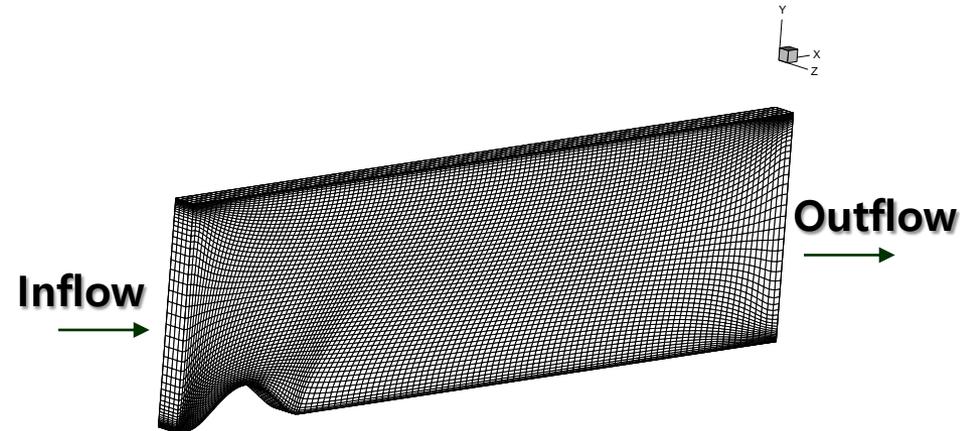
hFlow™(Code Validation for Turbulence Scheme)

1. Validation from

- G. P. Almeida, D. F. G. Durao, M. V. Heitor
 Wake Flows Behind Two-Dimensional Model Hills
 Experimental Thermal and Fluid Science, Vol. 7, pp. 87-101, 1993

2. Calculation Conditions

Re = 6×10^6
 Tu_∞ = 3 %
 U_o = 2.147 m/s
 R = 56 mm
 h = 28 mm



3. Numerical Scheme

Convection Scheme - 2nd Order Upwind

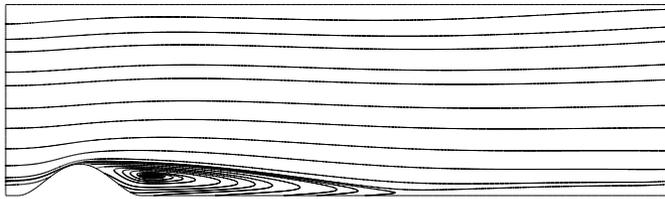
Flux Splitting Scheme - Liou et al. (1990)

Pressure & Velocity Linkage -SIMPLE

Turbulence Model - k-w (Wilcox, 1994)

4. Mesh generation

hFlowTM(Code Validation for Turbulence Scheme)



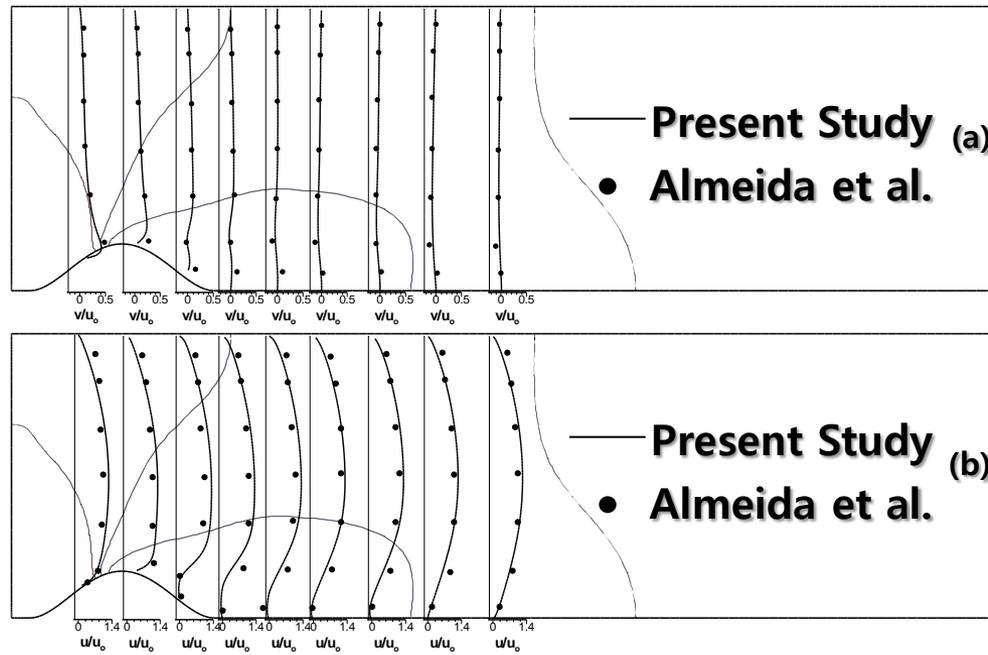
(a)



(b)

1. Field distribution of streamlines for a single hill.

(a) Numerical Calculation (b) Experimental Result



2. Vertical profiles of mean velocity characteristics over a single hill.

(a) Mean vertical velocity, U/U_0 (b) mean horizontal velocity, V/U_0

IV. Stress Analysis -sFlow-

sFlow™ (Stress Analysis Code)

1. Equilibrium Equation

$$(\lambda + G) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) + G \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \frac{\alpha E}{1 - 2\nu} \frac{\partial T}{\partial x} - X$$

2. Elastic Modulus

$$G = \frac{E}{2(1+\nu)} \quad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

3. Constitutive Equation

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} & \varepsilon_y &= \frac{\partial v}{\partial y} & \varepsilon_z &= \frac{\partial w}{\partial z} & e &= \varepsilon_x + \varepsilon_y + \varepsilon_z \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} & \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} & \gamma_{xy} &= \frac{1}{G} \tau_{xy} & \gamma_{yz} &= \frac{1}{G} \tau_{yz} & \gamma_{zx} &= \frac{1}{G} \tau_{zx} \\ \sigma_x &= \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \varepsilon_x & \sigma_y &= \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \varepsilon_y & \sigma_z &= \frac{\nu E}{(1+\nu)(1-2\nu)} e + \frac{E}{1+\nu} \varepsilon_z \end{aligned}$$

4. Principal Stress

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0$$

5. Invariant

$$I_1 = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_y \sigma_z + \sigma_z \sigma_x + \sigma_x \sigma_y - \tau_{yz}^2 - \tau_{xz}^2 - \tau_{xy}^2 = \sigma_2 \sigma_3 + \sigma_3 \sigma_1 + \sigma_1 \sigma_2$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix} = \sigma_1 \sigma_2 \sigma_3$$

6. Failure criteria developed by Von Mises and Hencky

$$\begin{aligned} \tau_0 &= \left(\frac{1}{9} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 + 6\tau_{xz}^2 + 6\tau_{yz}^2 \right] \right)^{1/2} \\ &= \frac{\sqrt{2}}{3} Y \end{aligned}$$

- Y : Tensile yield stress of the material as determined from a standard tension test

sFlow™(Code Validation for Stress Analysis)

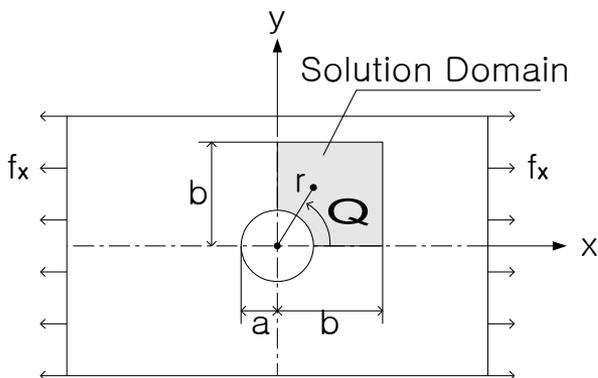
1. Validation from

- In plate with a circular hole,
Timoshenko and Goodier(1970)

$$\sigma_{xx} = f_x \left[1 - \frac{a^2}{r^2} \left(\frac{3}{2} \cos 2\theta + \cos 4\theta \right) + \frac{3 a^4}{2 r^4} \cos 4\theta \right]$$

$$\sigma_{yy} = f_x \left[-\frac{a^2}{r^2} \left(\frac{1}{2} \cos 2\theta - \cos 4\theta \right) - \frac{3 a^4}{2 r^4} \cos 4\theta \right]$$

$$\sigma_{xy} = f_x \left[-\frac{a^2}{r^2} \left(\frac{1}{2} \sin 2\theta + \sin 4\theta \right) + \frac{3 a^4}{2 r^4} \sin 4\theta \right]$$



3. Schematic diagram of Calculation domain.

2. Calculation Conditions

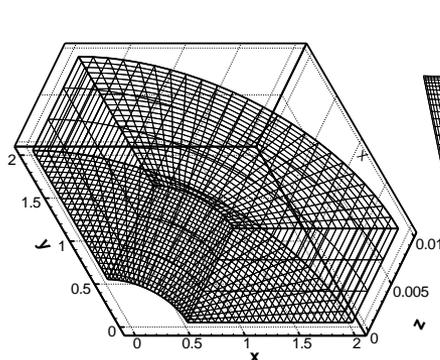
$$f_x = 10^4 \text{ Pa}$$

$$E = 10^7 \text{ Pa}$$

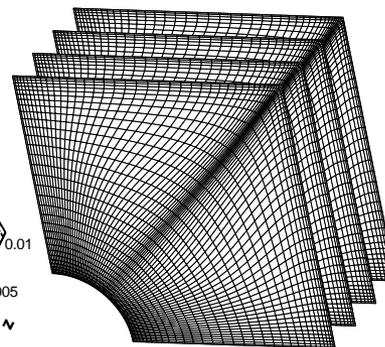
$$a = 0.5 \text{ m}$$

$$b = 2 \text{ m}$$

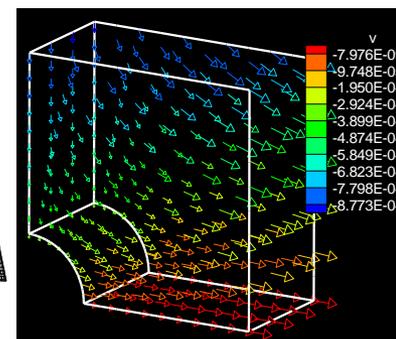
$$\nu = 0.3$$



(a)



(b)

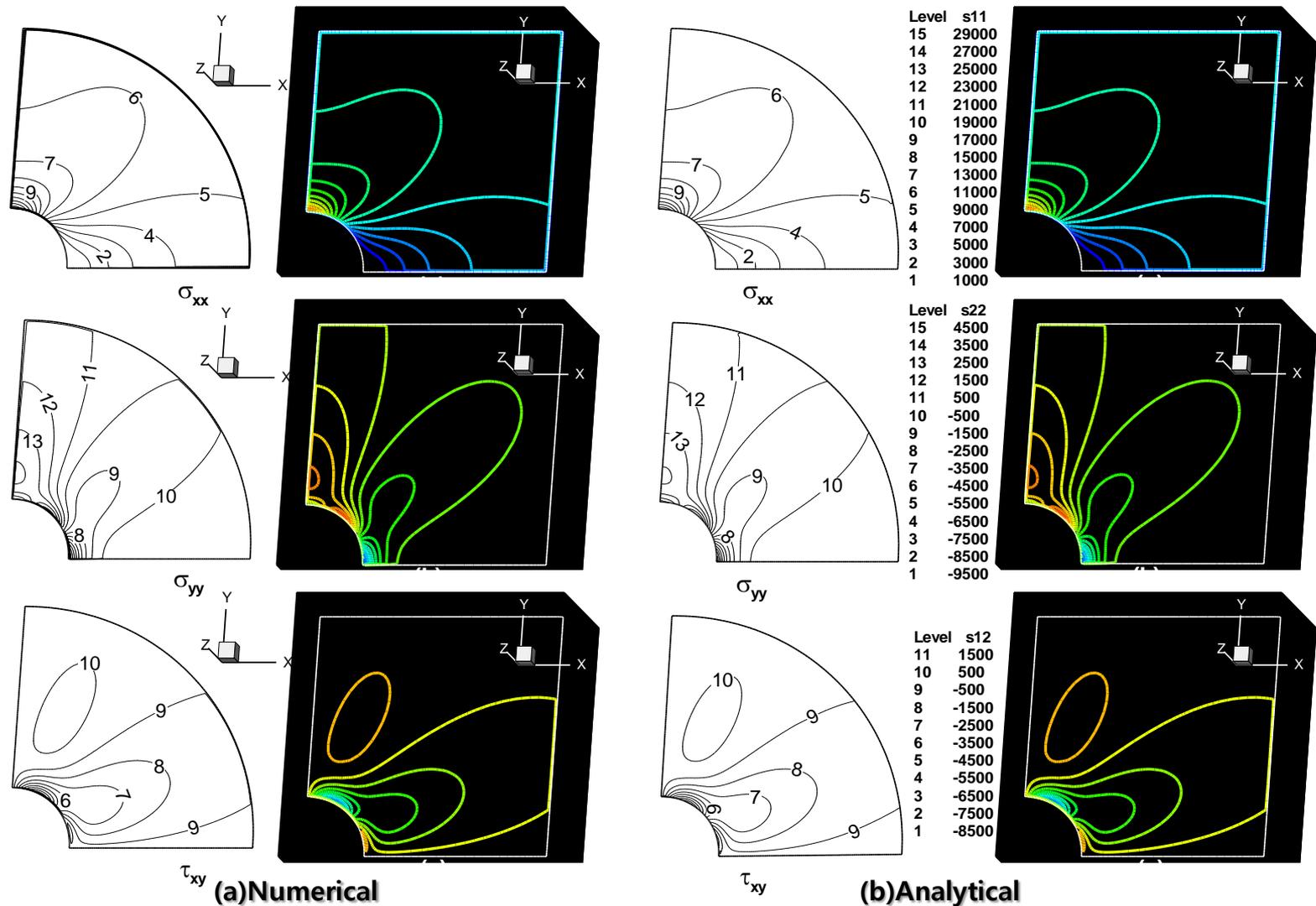


(c)

4. Computational mesh and displacement vector distribution.

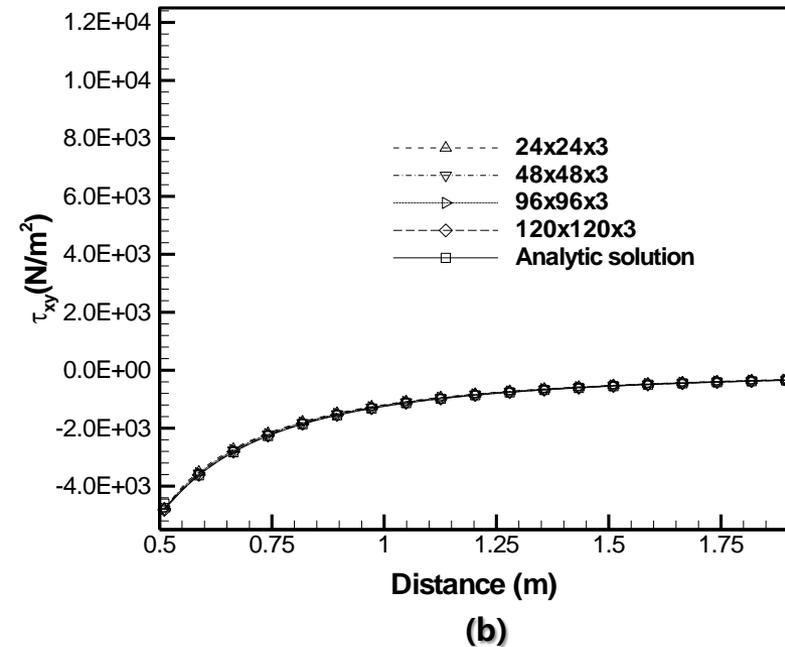
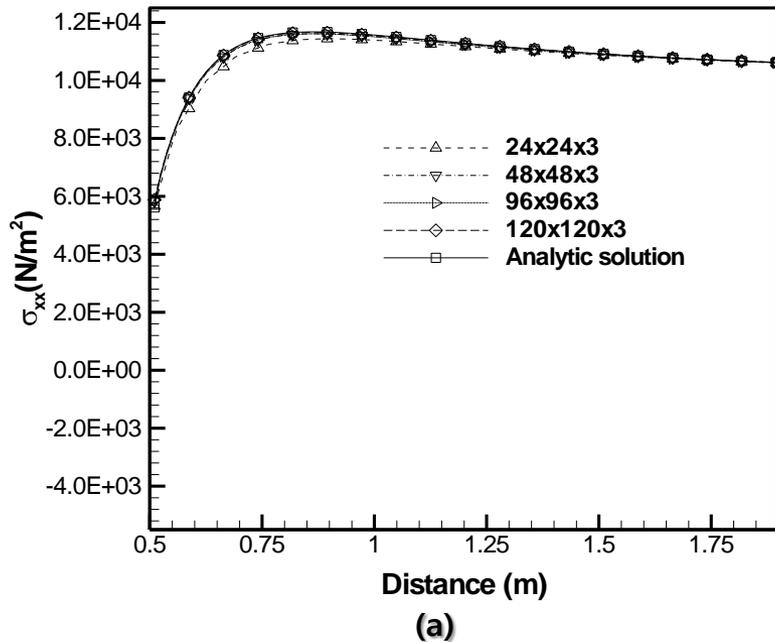
(a)Round edge, (b)Rectangular edge,
(c)Displacement vector

sFlow™(Code Validation for Stress Analysis)



1. Comparison of (a)numerical and (b)analytical principal stress fields.

sFlow™(Code Validation for Stress Analysis)



1. Comparison of numerical and analytical stress distributions at $\theta=45^\circ$ with grid dependency.

(a) x direction normal stress, (b) shear stress

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